Introduction for students:

At the end of this module you should have increased your ability to;

- Recognise and continue patterns in numbers
- Recognise, describe, compare and use rules that link sets of numbers
- Use a letter to represent a variable quantity
- Use basic algebraic conventions to represent rules
- Recognise and apply linear functions
- Recognise equivalent linear functions
- Graph linear functions understanding gradient and y intercept
- Solve linear equations

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**Activity A:**

Materials required: Coloured disks (5 x one colour, 25 x another colour), matches or toothpicks, graph paper.

**Patterns and Possibilities**

James has decided to build a path in his garden using circular paving slabs. Two paths are shown here. The first is of length 1 and the second is of length 2.

**Task 1:** Explain why James said the lengths were 1 and 2 rather than 3 and 5. What was he counting?

**Task 2:** Make a path of length 5 and record how many slabs of each colour that James would need to make it.

**Task 3:** James decided to make a table of numbers of slabs so that he could refer to it at any time. Complete the table here showing the number of slabs.

<table>
<thead>
<tr>
<th>Dark slabs</th>
<th>d</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light slabs</td>
<td>1</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Task 4:** Write about any patterns you see in the table.
Task 5: James wondered if he could be sure if the pattern he saw would continue. Referring to the original physical paving path rather than the table, explain why James can be confident that it will. Discuss your answer with your group.

Anne is using pieces of wood to make her path and will fill the interiors with gravel. Some of her work is shown in the diagram here.

Task 6: Using the matchsticks (or just on paper) create patterns of length 1, 3 and 5. For each of the patterns, including the two shown right, count the number of pieces of wood needed to make them. Record your results in a table like the one in task 3 up to a length of six squares.

Task 7: Describe any patterns in the table.

To represent the data Anne constructed a graph like the one started here.

Task 8: Consider carefully what scale to use on the axes and then use graph paper to copy and complete Anne’s graph for lengths from 1 to 6. Join the dots.

Note: Keep your graph for later use.
Task 9: Describe the graph.

Task 10: Anne wondered if she should have left the graph as a series of dots, rather than joining the dots. Comment on this. (Hint: Think about the values between the dots and what a length of 2.5 means in the ‘real’ world.)

Task 11: Use any patterns you found in the table or the graph to calculate the number of pieces of wood that would be required for:

(a) A path length of 7 squares
(b) A path length of 23 squares

Task 12: Explain how you got these answers.

Task 13: What length paths could be made with:

(a) 31 pieces of wood
(b) 46 pieces of wood

Task 14: Explain how you got these answers.

Task 15: If Anne had a total of 60 pieces of wood:

(a) What is the longest length path she could make?
(b) How many pieces of wood would be left over?
Activity B:
Materials required: Graph paper

Toothpick Animals

Swathi was making small “animals” for a children’s party by poking toothpicks into apples (see above right) and wanted to check whether she had enough toothpicks for the number of apples her mother had left her.

Task 1: She created a table to record the number of apples and the number of toothpicks used, for 1 to 6 apples. Fill in the table here for Swathi and then draw a graph of the data. Keep the graph for later use.

<table>
<thead>
<tr>
<th>Apples (a)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toothpicks</td>
<td>t</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Task 2: Write a rule in words that gives the number of toothpicks for any number of apples:

Number of toothpicks =

Task 3: Use your rule to find the number of:

Toothpicks required for 26 apples

Apples required to use 400 toothpicks

Jessica runs a baby-sitting business. She charges $5 for the first half hour, and then $3 for each additional half hour. To make it easier for her to quote over the phone, Jessica made a table.

Task 4: Complete the table.

<table>
<thead>
<tr>
<th>No of ½ hours (t)</th>
<th>Cost (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5</td>
</tr>
<tr>
<td>2</td>
<td>$8</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Task 5: Draw a graph showing number of half hours on the horizontal axis (from 1 to 10) and cost on the vertical axis. Keep the graph for later use.
**Task 6:** Jessica felt that there was a rule connecting the cost with the number of half hours. Use the table or graph to find the rule and write it here:

Cost ($) =

**Task 7:** Use your rule to find:

- The cost of babysitting for 8 half-hours
- The cost of babysitting for 7½ hours (be careful!)
- The time that parents can be away if they have only $20

There is a connection between the table of values in these activities and the rule. This connection is related to the difference pattern in each table.

<table>
<thead>
<tr>
<th>Apples (a)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toothpicks (t)</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td><strong>Difference pattern</strong></td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

**Task 8:** Explain how to get the difference pattern.

**Task 9:** How is the difference pattern related to the rule for this table?

The values in this table would graph as a line like those in Activity A. For this reason the rule is known as a linear function.

Using the variables given, the function would be written: $t = 5 \times a$

**Task 10:** How is the difference pattern related to the function?

The difference pattern from the second table can also be related to the rule and the function.

**Task 11:** Complete the difference pattern for this table.

<table>
<thead>
<tr>
<th>No of ½ hours (t)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in dollars (c)</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td><strong>Difference pattern</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Task 12:** How is this difference pattern similar to the one in the earlier table?

The graph from task 5 shows that this is also a linear function. In a linear function, the differences are always the same throughout the table. This is known as a constant difference.

**Task 13:** Complete the linear function below using the variables given in the table. Remember that the difference will help.

\[ C = \underline{\ \ \ \text{x} \ + \underline{\ \ \ \text{}}} \]

George is making jewellery to sell at the local markets. His creations are bracelets made from interlocking silver rings. The interlocking large silver rings are locked together by smaller enamelled rings. Bracelets of length 1 and 5 are shown here.

**Task 14:** George also decided to create a table in which he could record the number of enamelled rings and the number of silver rings used. Complete the table shown below for George and then draw a neat labelled graph. Keep the graph for later use.

<table>
<thead>
<tr>
<th>Enamel rings (e)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver rings (s)</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

**Task 15:** Is the function for this table linear? How do you know?

**Task 16:** Complete the function below for this table:

\[ S = \underline{\ \ \ \text{x} \ + \underline{\ \ \ \text{}}} \]

**Task 17:** Use your function to find the number of:

- Silver rings required for 17 enamelled rings
- Enamelled rings required to link 340 silver rings
James is making a sand pit for his younger brother to play in. He is using short wooden planks to make the sides. Pits of size 1, 2 and 3 are shown here.

Task 18: James created a table in which he could record the sizes of possible pits and the number of planks needed to make them. Fill in the table below for James and then draw a neat labelled graph.

<table>
<thead>
<tr>
<th>Size of pit (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planks (p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference pattern</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Task 19: Is the function for this table linear? How do you know?

Task 20: Complete the function below for this table:

\[ p = \_] \times s + \_ \]

Task 21: Use your function to find:

- The number of planks required for a pit of size 11.
- The size of the pit that can be built using 300 planks.

Task 22: In the activity above s is called the independent variable and p is the dependent variable. Why?

Task 23: In your group make up one situation similar to the ones that we have seen so far.

- Begin by explaining the situation.
- Draw up a table and collect data from your situation to fill it.
- Graph your data using suitable scales on the axes.
- Define variables for your quantities and find a rule for your situation.
Activity C:

Stepping Out Smartly

In designing stairs and ramps the slope or gradient is important for safety and comfort. Hills also have a gradient, too steep and cars find it difficult to ascend or unsafe to park.

The regulations about gradient that will apply in this activity are:
- A wheelchair ramp should not have a slope of more than 0.15
- A car may be safely parked on a hill if the slope is not more than 1 in 4 (0.25)
- When building stairs, the slope should not be greater than 7 in 8 (0.875)
- A ramp for walking should not have a slope of more than 0.325

A slope is measured by looking at the gradient of the sloping line. The equation which gives the gradient is:

\[
\text{gradient} = \frac{\text{rise}}{\text{run}} \quad \text{i.e.} \quad \frac{\text{vertical length}}{\text{horizontal length}}
\]

Task 1: Use this equation to find the gradient in these diagrams.
**Task 2:** According to the regulations which of these slopes would be safe for use with?

A wheelchair?  
A parked car?  
Stairs?  
A pedestrian ramp?

**Task 3:** Decide which of the following driveways are too steep to allow a car to be safely parked:

(a) James’ driveway which has a run of 12.3 metres for a rise of 2.8 metres.

(b) Jessica’s driveway which has a rise of 1.8 metres and a run of 5.6 metres.

(c) Jocasta’s driveway which rises 98 cm during a run of 6.25 metres

**Task 4:** The diagram below shows a verandah at a school. A ramp must be built so that a student in a wheelchair will have easy access. Will a ramp that extends 7 metres out from the edge of the verandah satisfy the safety regulations?

![Diagram of a verandah with 90 cm drop]

**Task 5:** What is the shortest ramp that can be fitted above which would still satisfy the regulations?
Task 6: To safely lean a ladder against a wall its **gradient** should be between 3.5 and 6. Is the ladder shown here safe?

Task 7: The base of another ladder which is 5 metres long is leaning against a wall so that the base is 1.2 metre out from the bottom of the wall. Is this a safe situation? **Hint:** You may need to consider how high up the wall the ladder extends. This may be done with a scale drawing.

Task 8: If a ladder is to reach 5.4 metres up a wall, what is the longest distance that the base can be from the bottom of the wall?

One of Australia's endangered species is the Mahogany Glider (Petaurus gracilis). This glider grows from the size of a grain of rice to a maximum weight of 500g and a maximum length from snout to tail-tip of 600mm.

The Mahogany glider is capable of gliding up to 50m. It has a glide ratio of 2:1. That is for every 2m travelled horizontally the glider drops 1m in height.

Task 9: What is the **gradient** of the Mahogany Glider's glide path?

Task 10: How high would a glider need to start if it was to glide the maximum 50m?

Task 11: If a glider takes off from a tree at height of 15m, what horizontal distance could it glide for?

Task 12: If a glider lands 17m from a tree after a glide how high in the tree did it take off from?
Activity D:
Materials required: Graphics calculator

Graphs of Linear Functions

In this activity you will investigate the relationship between the equation of a line and its graph. At the end of this activity you should be able to explain how m and c affect the appearance of the graph of the linear equation \( y = mx + c \).

**Task 1:** The first task is to set up the axes so that they will show the lines well. On a Casio or a Texas Instruments you should set the x axis to run from \(-10\) to \(10\) and the y axis from \(-6\) to \(6\). On an HP calculator you should use the same x axis but set the y axis to run from \(-5\) to \(5\).

**Task 2:** Whichever sort of calculator you are using you should also set the grid to "on" so that you can more easily use "rise over run" methods to find gradients.

**Task 3:** When you finish doing this, enter the equation \( x + 1 \) into the equation window and graph it. You should find that your graph looks like one of the ones shown below.

As you can see it should not make any real difference which machine you are using and your teacher will show you how to set the axes and how to enter the equations to be graphed. Note that even though the equation is correctly written as \( y = x + 1 \), all the calculators require that you only enter the \( x + 1 \) part of it.

When investigating something in which two values can change, in this case the values of \( m \) and \( c \), it is best to only change one at a time. During this task you should keep the value of \( m \) as \( 1 \) and change the value of \( c \).

**Task 4:** In the symbolic entry view of your calculator, enter the equations \( y = x - 2, y = x, y = x + 1 \) and \( y = x + 3 \). Graph the results and write down any observations which you can make about the four graphs which result. Your observations should include the gradients (use "rise over run") and the y-intercepts of each line. In what ways are the values of \( c \) in the equations related to the graphs?
Task 5: Now change the value of \( m \) to 2 and see how this changes the graphs. In the symbolic entry view of your calculator, enter the equations \( y = 2x - 2, y = 2x, y = 2x + 1 \) and \( y = 2x + 3 \). Graph the results and again write down any observations which you can make about the four graphs which result.

Task 6: Look carefully at your two sets of observations and compare them. In what ways are the values of \( m \) and \( c \) in the two sets of equations related to the gradients and y-intercepts of the graphs?

Task 7: The next task is to vary the value of \( m \) while keeping \( c \) constant. In the symbolic entry view of your calculator, enter the equations \( y = x, y = 2x, y = 3x \) and \( y = 4x \). In your own words, explain how the slope changes as the value of \( m \) increases.

Task 8: Predict what will happen for decimal values of \( m \) and check your predictions. For example, what do you think the graph of \( y = 0.5x \) will look like? Which of \( y = 0.25x + 1 \) and \( y = 0.1x + 1 \) will be steeper? In your own words, what is your conclusion?

Task 9: Predict what the graphs of \( y = 3x - 2 \) and \( y = 0.5x + 1 \) should look like and then graph them to see if your predictions are correct. If you are not then discuss your understanding with your neighbour.

Task 10: When investigating equations like this one, students often forget to include negative values and decimal values. What happens when the values of \( m \) are negative? Hint: Try comparing similar but opposite values. For example, \( y = 2x \) with \( y = -2x \) and \( y = x + 3 \) with \( y = -x + 3 \).

Task 11: Discuss your final conclusions with your neighbour and then write a summary of the effects of the coefficients \( m \) and \( c \) in the equation \( y = mx + c \). Your explanation should include how to predict the gradient and the \( y \) intercept from the equation.
Activity E:
Materials required: Graphics calculator

Using Formulae

Formulae are used in many diverse areas, from calculating the energy given out by a heater to finding the frequency of a guitar string.

The perimeter of a square can be calculated using the formula:\[ P = 4l \] where \( P \) represents the perimeter and \( l \) the length of a side. In this formula and other parts of algebra, the lack of a symbol between the 4 and the \( l \) indicates multiplication.

Task 1: Use this formula to find the perimeter of squares with side lengths of:
(a) 3cm  (b) 2.73m  (c) 13.6mm.

In this task you were substituting values into a formula. This is known as substitution.

Task 2: Now reverse the direction of the calculation to find the side length required for squares with perimeters of:
(a) 40cm  (b) 36mm  (c) 2.7m

In this task you were finding values that gave a specific result from a formula. This is known as equation solving.

Ron has a pen friend in the United States. Recently his friend referred to a “nice day with a temperature of 80°F” and Ron was very puzzled because this seemed too hot to be possible. His friend explained that the U.S. used the Fahrenheit scale to measure temperature rather than the Celsius scale as they do in Australia.

Task 3: The conversion from Celsius to Fahrenheit is done using the formula: \[ F = 1.8C + 32 \]. (F is °F, C is °C)

Use the formula and substitution to complete this table:

<table>
<thead>
<tr>
<th>°C</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>°F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Task 4: How can you tell this is a **linear formula**?

Task 5: Use the formula and **substitution** to convert Celsius temperatures of 15°, 25°, 30° and 35° to Fahrenheit.

Task 6: Use trial and adjustment or any other method to find the temperature in Australia which corresponds to “hitting the century” (100°F) in the U.S.

In banking and finance interest is earned on money invested. The amount of this interest depends on the interest rate, the time the money is invested for and the amount of money invested.

If $2500 is invested at an interest rate of 4.5% per annum (per year) the amount of simple interest (S.I.) earned will depend on the time (T).

The **formula** for this is:  
\[ S.I. = 2500 \times 0.045 \times T \]

Task 7: Find the amount of simple interest earned for these time periods using **substitution**:

(a) 1 year  (b) 2 years  (c) 5 years

Task 8: Enter the **function** for Simple Interest into the **function** mode of your calculator.

Task 9: The amounts of interest for various numbers of years can now be found in the number mode. Set this to show whole years at first and use it to find the simple interest earned for:

(a) 7 years  (b) 25 years  (c) 10 years

Task 10: Work out how long you would have to invest the money for it to earn $1000 interest.
In task 10 you solved this equation: \(2500 \times 0.045 \times T = 1000\)

**Task 11:** Enter this equation into the solve mode of your calculator.

**Task 12:** Now use the calculator to solve the equation.

**Task 13:** Enter the equation \(1.8C + 32 = 100\) into the solve mode to find exactly what temperature in degrees Celsius is equivalent to \(100^\circ\) Fahrenheit.

**Task 14:** A quick method of converting Celsius to Fahrenheit temperatures is “double it and add thirty”. Explain why this gives reasonable results.

**Task 15:** Use this method to find the approximate value for \(20^\circ\)C converted to Fahrenheit.

**Task 16:** What is the true conversion of \(20^\circ\)C into Fahrenheit?

The formula for percentage error is: \[
%\text{error} = \frac{(\text{value} - \text{true value})}{\text{true value}} \times 100
\]

**Task 17:** Use this formula to find the percentage error in the quick conversion of \(20^\circ\)C to degrees Fahrenheit.

**Task 18:** Comment on the accuracy of the quick conversion method for converting \(20^\circ\)C.

**Task 19:** George has decided that the greatest acceptable error in using the quick conversion method is an error of 5% above or below the true value. Over what range of temperatures will the approximation be acceptable?

Note: This is not an easy question – you may need to discuss strategies for answering it with your teacher.
Activity F:

Investigating Lockers

Isaac Newton Senior High School has just been renovated and contains exactly 1000 brand new full height lockers available for the students. The lockers are numbered 1 to 1000.

Mr Jackson, the new mathematics teacher, decided to try an experiment with his maths class. He lined up the students in his class at the beginning of the corridor of lockers. Each student is to walk down the corridor one after the other.

The first student is to open all the locker doors. He then goes to the end of the line to wait for his next turn.

The second student follows and closes all the locker doors which have even numbers.

The third student then follows and changes the state of every third locker door. ie. any locker with a number which is a multiple of three. Mr Jackson explained that by changing the state, it meant you had to open a door that is closed and close a door that is open.

The fourth student now changes the state of all the locker doors that are numbered with a multiple of four during his trip down the thousand locker doors. This continues until ten students have made their trip down the corridor of lockers.

After the first ten students have made their trip, Mr Jackson posed the following question: “If we keep on doing this until this trip has been made by 1000 students, which locker doors will be open?”

Task 1: If you have a theory already on what the answer might be, record it below.

You may well be looking at “1000 lockers” and thinking that there is no way you can solve this problem. If so, the best thing to do is to start with a smaller number. In this case, we will try just 17 lockers.
**Task 2:** In the table below, O = Open and C = Closed. The state of the 17 lockers are shown for the first two students and for part of the third. Complete the table for the 17 students.

<table>
<thead>
<tr>
<th>Locker No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
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<tr>
<td>Student 1</td>
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<tr>
<td>Student 2</td>
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<td>C</td>
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</tr>
<tr>
<td>Student 3</td>
<td>O</td>
<td>C</td>
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<td>Student 14</td>
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<td>Student 16</td>
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<td>Student 17</td>
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</tr>
</tbody>
</table>

**Task 3:** List the numbers of the lockers which are open at the end of the 17th student’s run. Can you now see a pattern in the ones that are open? If so, record it.

Lockers open:

Pattern found:

**Task 4:** If you have now spotted the pattern then what is the answer to Mr Jackson’s original question? How many of the 1000 lockers will be open when the 1000th student has passed through?

**Task 5:** The question you should now try to answer is: “Why does it happen?”

**Hint:** It has something to do with the number of factors that the different numbers have. In one column, try writing down all the factors, in order, for a couple of the locker numbers which were closed. Now, in a second column, do the same for some of the lockers that were open. Look at how many factors there are in each. You should try to explain in a logical fashion why this causes the pattern to occur, in terms of opening and closing.
**Student Recording:**

Write at least two pieces of information about each of these concepts that you have explored in earlier lessons. Then try to give an example relating to each. Use diagrams where it helps.

<table>
<thead>
<tr>
<th>rule:</th>
<th>gradient (slope):</th>
</tr>
</thead>
<tbody>
<tr>
<td>difference pattern:</td>
<td>y intercept:</td>
</tr>
<tr>
<td>constant difference:</td>
<td>equation:</td>
</tr>
<tr>
<td>function:</td>
<td>linear equation:</td>
</tr>
<tr>
<td>linear function:</td>
<td>formula:</td>
</tr>
<tr>
<td>variable:</td>
<td>substitution:</td>
</tr>
<tr>
<td>independent/dependent variable</td>
<td>solve/solving:</td>
</tr>
</tbody>
</table>
Notes

Linear Functions

A function is a rule that determines one set of values from another set.

A linear function is one that will graph to give a straight line.

A linear function contains two variables, an independent variable and a dependent variable. In the table here the independent variable is \( a \) and the dependent variable is \( b \).

A table of values can be generated from a function by substituting successive values into the independent variable. A linear function is always formed by multiplying by a number and then adding another, although one or both of these numbers may be zero.

If the function is entered into a graphics calculator the "number" mode will contain a table of values.

It is possible to tell from the table of values whether a pattern is linear or not by examining the difference pattern as shown in the table below.

<table>
<thead>
<tr>
<th>( a )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>7</td>
<td>11</td>
<td>15</td>
<td>19</td>
<td>23</td>
<td>27</td>
</tr>
</tbody>
</table>

| Difference pattern | 4 | 4 | 4 | 4 | 4 | 4 |

A constant difference identifies a linear function. The constant difference is also the same as the number in the rule which is multiplied by the independent variable.
Gradient and Y Intercept of a Line

The general function for a straight line is \( y = mx + c \), where \( m \) and \( c \) are numbers which can have any value including fractions, decimals and negatives. This may not be obvious if the rule is written in some other form than \( y = mx + c \).

The \( m \) value represents the gradient or slope of the line.

This is defined as the rise divided by the run. i.e. \( \frac{\text{rise}}{\text{run}} \)

The constant difference and the gradient are always equal for a linear function.

Lines which have the same gradient are parallel.

The \( c \) value represents the y intercept of a line. This is the place where the line crosses the vertical axis of the graph.

This can be found by either extending the line back until it crosses the vertical axis, or by extending the table backwards to where the independent variable is zero.

The function shown here is \( b = 4a + 3 \).

The \( m \) value is 4.

This means the gradient is 4.

The graph shows that the rise over run works out to be 4 for the line.

The \( c \) value is 3.

This means the y intercept is 3.

The graphs shows that the line crosses the vertical axis at \( b = 3 \).

This matches the table of values where the constant difference is 4. If the table were extended backward to where \( a = 0 \) by subtracting 4 from the \( b = 7 \) value then we get the y intercept of 3.
**Exercise 1: Completing Tables of Values**

By looking at the diagrams complete the table of values for these situations and decide whether they are linear or not.

1. 

   | diagram | d | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
   | matches | m | 3 | 5 |   |   |   |   |   |

2. 

   | diagram | d | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
   | matches | m | 4 |   |   |   |   |   |   |

3. 

   | diagram | d | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
   | spots   | s |   |   |   |   |   |   |   |

4. 

   | diagram | d | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
   | spots   | s |   |   |   |   |   |   |   |
5. 

<table>
<thead>
<tr>
<th>white circles</th>
<th>w</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>black circles</td>
<td>b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. 

<table>
<thead>
<tr>
<th>diagram</th>
<th>d</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>squares</td>
<td>s</td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

7. 

<table>
<thead>
<tr>
<th>diagram</th>
<th>d</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>squares</td>
<td>s</td>
<td></td>
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</tr>
</tbody>
</table>

8. 

<table>
<thead>
<tr>
<th>diagram</th>
<th>d</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>spots</td>
<td>s</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
**Exercise 2: Tables of Values**

For these linear functions complete the tables of values by substitution or using the table mode on a graphics calculator.

1. \( b = a + 3 \)

<table>
<thead>
<tr>
<th>a</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. \( d = 3c - 5 \)

<table>
<thead>
<tr>
<th>c</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. \( y = \frac{x}{4} + 5 \)

<table>
<thead>
<tr>
<th>x</th>
<th>-8</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>7.5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. \( f(x) = 10 - 2x \)

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>7.5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. \( f(x) = 2(x + 3) \)

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-3</th>
<th>0</th>
<th>1</th>
<th>5</th>
<th>7.5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Why don’t these tables show a constant difference?
### Exercise 3: Identifying Function Types

Complete each table then identify which of the tables are linear.

1. | a | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

2. | c | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

3. | a | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

4. | c | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

5. | a | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

6. | c | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>10</td>
<td>7</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

7. | a | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>-3</td>
<td>-7</td>
<td>-11</td>
<td>-15</td>
</tr>
</tbody>
</table>

8. | c | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

9. | a | 1 | 2 | 3 | 4 | 5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>-3</td>
<td>2</td>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

10. | c | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|
    | d | 30 | 15 | 10 | 7.5 |

11. | a | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|
    | b | 2 | 3 | 5 | 8 |

12. | c | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|
    | d | 1 | 1 | 2 | 3 |

13. | a | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|
    | b | 21 | 33 | 45 | 57 |

14. | c | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|
    | d | 6 | 0 | -6 | -12 |

15. | a | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|
    | b | -5 | -4 | -2 | 1 |

16. | c | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|
    | d | 1 | 8 | 27 | 64 |
### Exercise 4: Finding Functions from Tables

Find the **linear functions** for these tables of values.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>b</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>d</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>f</td>
<td>3</td>
<td>6</td>
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<tr>
<td>y</td>
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<tr>
<td>x</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<tr>
<td>f(x)</td>
<td>4.5</td>
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<td>4</td>
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</tr>
<tr>
<td>f(x)</td>
<td>-5</td>
<td>-7</td>
<td>-9</td>
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<table>
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<td>1</td>
<td>2</td>
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<td>4</td>
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</tr>
<tr>
<td>f(x)</td>
<td>19</td>
<td>16</td>
<td>13</td>
<td>10</td>
<td>7</td>
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<table>
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<tr>
<td>x</td>
<td>1</td>
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<td>4</td>
<td>5</td>
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<td>f(x)</td>
<td>7</td>
<td>11</td>
<td>15</td>
<td>19</td>
<td>23</td>
</tr>
</tbody>
</table>
Exercise 5: Drawing Graphs

Draw graphs of the linear functions in these tables by plotting points to represent the values. Use axes as shown in the diagrams.

1. \[
\begin{array}{c|c|c|c|c|c}
    \text{x} & 1 & 2 & 3 & 4 & 5 \\
    \text{y} & 3 & 5 & 7 & 9 & 11 \\
\end{array}
\]

2. \[
\begin{array}{c|c|c|c|c|c}
    \text{x} & 1 & 2 & 3 & 4 & 5 \\
    \text{y} & 7 & 15 & 23 & 31 & 39 \\
\end{array}
\]

3. \[
\begin{array}{c|c|c|c|c|c}
    \text{x} & -2 & -1 & 0 & 1 & 2 \\
    \text{y} & 11 & 8 & 5 & 2 & -1 \\
\end{array}
\]

4. \[
\begin{array}{c|c|c|c|c|c}
    \text{x} & -5 & -1 & 0 & 1 & 5 \\
    \text{y} & 2 & -2 & -3 & -4 & -8 \\
\end{array}
\]

5. Describe the shape of the four graphs.
Exercise 6: Sketching Graphs

The graphs in this exercise should be sketched from a graphics calculator.

1. Draw graphs of $y = x + 2$, $y = 2x + 2$ and $y = 4x + 2$ on the same axes.

2. What effect does the change from $1x$ to $2x$ to $4x$ have on the graphs?

3. Draw graphs of $y = 3x - 1$ and $y = 3x + 5$ on the same set of axes.

4. Comment on what you notice about the two graphs.

5. How could you have predicted this from the functions?

6. Draw graphs of $y = -2x + 3$ and $y = -2x - 1$ on the same set of axes.

7. Comment on what you notice about the two graphs.

8. What effect does the $-2$ from each function have?

9. Draw graphs of $y = 0.5x + 3$ and $y = \frac{x}{2} + 6$ on the same set of axes.

10. Why are these two graphs parallel?

11. Draw graphs of $y = 2x + 2$, $y = 2x + 5$ and $y = 2x - 3$ on the same axes.

12. What effect does the change from $+2$ to $+5$ to $-3$ have on the graphs.

13. The graphs here are for the same function. Explain why they appear to be different.
Exercise 7: Gradient & Y intercept

PART A: For each graph below, find the gradient and the y intercept.

1. Gradient _____ Y intercept _____

2. Gradient _____ Y intercept _____

3. Gradient _____ Y intercept _____

4. Gradient _____ Y intercept _____
Part B: For each of the functions shown below, give the gradient and the value of the y intercept.

<table>
<thead>
<tr>
<th>Function</th>
<th>Gradient</th>
<th>Y intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. $y = -3x + 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. $y = 5 + 2x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. $y = \frac{x}{3} - 5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. $y = 6 - 0.5x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. $2x + y = 7$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Part C: For each of the graphs below and on the next page, find the gradient and the y intercept and hence the function.

10. (a) gradient _____ (b) y intercept _____ (c) function _____

11. (a) gradient _____ (b) y intercept _____ (c) function _____
12. (a) gradient _____  
(b) y intercept _____  
(c) function _____

13. (a) gradient _____  
(b) y intercept _____  
(c) function _____

14. (a) gradient _____  
(b) y intercept _____  
(c) function _____

15. (a) gradient _____  
(b) y intercept _____  
(c) function _____
Exercise 8: Equivalent Functions

1. **Substitute** \( p = 3 \) into each of these functions to find the value of \( q \).
   
   \( \begin{align*}
   (a) & \quad q = p + 6 \\
   (b) & \quad q = 6 + 1p \\
   (c) & \quad q = 6p + 1
   \end{align*} \)

2. Write down the equivalent functions from question 1.

3. **Substitute** \( x = 5 \) into each of these functions to find the value of \( y \).
   
   \( \begin{align*}
   (a) & \quad y = 2x + 4 \\
   (b) & \quad y = 4 + 2x \\
   (c) & \quad y = 4x + 2 \\
   (d) & \quad y = 2(x + 4) \\
   (e) & \quad y = 2(x + 2) \\
   (f) & \quad y = 2x + 2
   \end{align*} \)

4. Write down the equivalent functions from question 3.

5. **Substitute** \( m = 4 \) into each of these functions to find the value of \( n \).
   
   \( \begin{align*}
   (a) & \quad n = 2 + 4m \\
   (b) & \quad n = 2 - 4m \\
   (c) & \quad n = 2m + 4 \\
   (d) & \quad n = 2(1 + 2m) \\
   (e) & \quad n = 2(2m + 1) \\
   (f) & \quad n = 2(2m + 2)
   \end{align*} \)

6. Write down the equivalent functions from question 5.

7. **Substitute** \( r = 6 \) into each of these functions to find the value of \( s \).
   
   \( \begin{align*}
   (a) & \quad s = 2r + 5 + r \\
   (b) & \quad s = 2r + 5r \\
   (c) & \quad s = 5 + 3r \\
   (d) & \quad s = 2(r + 5) \\
   (e) & \quad s = 3r + 5 \\
   (f) & \quad s = 2r + 1r + 5
   \end{align*} \)

8. Write down the equivalent functions from question 7.

9. **Substitute** \( v = (-2) \) into each of these functions to find the value of \( w \).
   
   \( \begin{align*}
   (a) & \quad w = 8v - 12 \\
   (b) & \quad w = 8(v - 12) \\
   (c) & \quad w = 4(2v - 3) \\
   (d) & \quad w = 12 - 8v \\
   (e) & \quad w = 2(4v - 6) \\
   (f) & \quad w = (-12) + 8v
   \end{align*} \)

10. Write down the equivalent functions from question 9.
Exercise 9: Solving Linear Equations

1. If \( s = 3 \) which of these equations are true:
   
   (a) \( s + 7 = 10 \) \hspace{1cm} (b) \( 2s = 23 \) \hspace{1cm} (c) \( 5 + 2s = 11 \)
   
   (d) \( 4(s + 2) = 14 \) \hspace{1cm} (e) \( 4(s + 2) = 20 \) \hspace{1cm} (f) \( 3s - 15 = (-6) \)

2. Find the value of \( x \) that makes each of these equations true:
   
   (a) \( x + 5 = 10 \) \hspace{1cm} (b) \( x - 4 = 8 \) \hspace{1cm} (c) \( 7 - x = 3 \)
   
   (d) \( 5 - x = (-4) \) \hspace{1cm} (e) \( x + 3 = (-2) \) \hspace{1cm} (f) \( x + (-4) = 6 \)

3. Find the value of \( y \) that makes each of these equations true:
   
   (a) \( 2y = 12 \) \hspace{1cm} (b) \( 3y = 18 \) \hspace{1cm} (c) \( 4y = (-16) \)
   
   (d) \( -y = 10 \) \hspace{1cm} (e) \( -3y = (-24) \) \hspace{1cm} (f) \( 0.5y = 10 \)

4. Find the value of \( k \) that makes each of these equations true:
   
   (a) \( 2k + 5 = 15 \) \hspace{1cm} (b) \( 3k - 2 = 16 \) \hspace{1cm} (c) \( 8 - 2k = (-4) \)
   
   (d) \( (-2) = 3k - 2 \) \hspace{1cm} (e) \( (-6) -3k = (-12) \) \hspace{1cm} (f) \( 9 - 6k = 0 \)

5. Solve the following equations:

   (a) \( 3a + 5a = 16 \) \hspace{1cm} (b) \( 5 + b = 2b - 2 \)
   
   (c) \( c - 5 = 19 - 2c \) \hspace{1cm} (d) \( 3d + 8 = 2 - 3d \)
   
   (e) \( 4(e + 3) = 36 \) \hspace{1cm} (f) \( 8 - 4(f + 2) = 16 \)
   
   (g) \( 3(2g - 6) = 12(g - 3) \) \hspace{1cm} (h) \( 6 - 4h = 2(h + 7) \)
   
   (i) \( i + 3(4 - i) = 2i - 8 \) \hspace{1cm} (j) \( -(k + 6) + 3k = 4k + 1 \)
Exercise 10: Translating Problems

1. Match the phrases on the left (a to h) with an equivalent algebraic expression on the right (1 to 8).

   (a) three more than x 1. x - 2
   (b) x times two 2. \( \frac{x}{2} \)
   (c) three reduced by x 3. x + 3
   (d) two less than x 4. 2x
   (e) the sum of x and two 5. x + 2
   (f) one half of x 6. \( \frac{x^2}{2} - 3 \)
   (g) two more than three times x 7. 3 - x
   (h) three less than the square of x 8. 3x + 2

2. Translate these phrases into algebraic expressions:
   (a) eight lots of a is increased by two
   (b) seven is subtracted from six times b
   (c) c is multiplied by five and then increased by two
   (d) d is increased by two and then multiplied by five

3. Write an equation for each of these sentences using x as the variable and then solve the equation:
   (a) Eight more than a number is equal to fifteen.
   (b) Three lots of a number is equal to twenty one.
   (c) Seven less than a number is equal to five.
   (d) One quarter of a number is equal to five.
   (e) If a number is doubled and then three is added the result is negative five.
Puzzle A:

Watch Eating Owls

To solve the riddle at the bottom of the page you need to match each linear function with its correct gradient \((m)\) and \(y\) intercept \((c)\).

The number of the function goes with the letter of the gradient and \(y\) intercept in the code boxes.

Function:

<table>
<thead>
<tr>
<th>Function</th>
<th>Gradient and Y Intercept</th>
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</thead>
<tbody>
<tr>
<td>1. ( y = 2x + 3 )</td>
<td>W. ( m = 2, c = (-6) )</td>
</tr>
<tr>
<td>2. ( y = x + 5 )</td>
<td>R. ( m = 0.5, c = (-3) )</td>
</tr>
<tr>
<td>3. ( y = \frac{x}{2} + 3 )</td>
<td>U. ( m = \frac{1}{2}, c = 3 )</td>
</tr>
<tr>
<td>4. ( y = 3x )</td>
<td>E. ( m = 1, c = 3 )</td>
</tr>
<tr>
<td>5. ( y = 2x - 6 )</td>
<td>D. ( m = 2, c = 3 )</td>
</tr>
<tr>
<td>6. ( y = -3x + 2 )</td>
<td>K. ( m = (-2), c = 3 )</td>
</tr>
<tr>
<td>7. ( y = 5x + 1 )</td>
<td>S. ( m = (-1), c = 3 )</td>
</tr>
<tr>
<td>8. ( y = 0.5x - 3 )</td>
<td>N. ( m = 3, c = 0.5 )</td>
</tr>
<tr>
<td>9. ( y = 4x + 7 )</td>
<td>E. ( m = 6, c = 2 )</td>
</tr>
<tr>
<td>10. ( y = x + 3 )</td>
<td>O. ( m = 1, c = 5 )</td>
</tr>
<tr>
<td>11. ( y = 6x + 2 )</td>
<td>S. ( m = 5, c = 1 )</td>
</tr>
<tr>
<td>12. ( y = \frac{x}{4} + 7 )</td>
<td>H. ( m = 3, c = 0 )</td>
</tr>
<tr>
<td>13. ( y = 3 - 2x )</td>
<td>T. ( m = 7, c = (-4) )</td>
</tr>
<tr>
<td>14. ( y = 3x + 0.5 )</td>
<td>L. ( m = (-1), c = 5 )</td>
</tr>
<tr>
<td>15. ( y = 7x - 4 )</td>
<td>I. ( m = 2, c = 6 )</td>
</tr>
<tr>
<td>16. ( y = 3 - x )</td>
<td>C. ( m = 4, c = 7 )</td>
</tr>
<tr>
<td>17. ( y = 2x + 6 )</td>
<td>E. ( m = (-3), c = 2 )</td>
</tr>
<tr>
<td>18. ( y = 5 - x )</td>
<td>C. ( m = \frac{1}{4}, c = 7 )</td>
</tr>
</tbody>
</table>

What happened to the owl who swallowed the watch?

| 16 | 4 | 11 | 15 | 3 | 8 | 14 | 6 | 1 | 9 | 18 | 2 | 12 | 13 | 5 | 17 | 7 | 10 |
## Puzzle B:

### Candles and Witches

Match the number of the formula with the letter that corresponds with the answer in the code key to complete the riddle below.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Known value</th>
<th>Unknown value</th>
<th>Code Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( a = 3b + 5 )</td>
<td>( b = 7 )</td>
<td>( a = ? )</td>
<td>B 15</td>
</tr>
<tr>
<td>2 ( b = 4d - 9 )</td>
<td>( d = 8 )</td>
<td>( b = ? )</td>
<td>W 72</td>
</tr>
<tr>
<td>3 ( e = 8f + 5 )</td>
<td>( e = -3 )</td>
<td>( f = ? )</td>
<td>E 26</td>
</tr>
<tr>
<td>4 ( g = 10 - 4h )</td>
<td>( h = 7.5 )</td>
<td>( g = ? )</td>
<td>H (-1)</td>
</tr>
<tr>
<td>5 ( i = 2.5j + 6.5 )</td>
<td>( j = 5 )</td>
<td>( i = ? )</td>
<td>K 20</td>
</tr>
<tr>
<td>6 ( k = 6 + 3l )</td>
<td>( l = -4 )</td>
<td>( k = ? )</td>
<td>R 23</td>
</tr>
<tr>
<td>7 ( m = -8 + 5n )</td>
<td>( m = -8 )</td>
<td>( n = ? )</td>
<td>C 14</td>
</tr>
<tr>
<td>8 ( o = 4p + (-10) )</td>
<td>( o = 50 )</td>
<td>( p = ? )</td>
<td>T (-20)</td>
</tr>
<tr>
<td>9 ( q = 4r - 5 )</td>
<td>( r = -2 )</td>
<td>( q = ? )</td>
<td>I 17</td>
</tr>
<tr>
<td>10 ( s = \frac{3}{4}t + 5 )</td>
<td>( t = 16 )</td>
<td>( s = ? )</td>
<td>A 19</td>
</tr>
<tr>
<td>11 ( u = 5(v + 7) )</td>
<td>( v = -3 )</td>
<td>( u = ? )</td>
<td>Y (-6)</td>
</tr>
<tr>
<td>12 ( w = 8(x - 9) )</td>
<td>( w = 40 )</td>
<td>( x = ? )</td>
<td>O (-13)</td>
</tr>
<tr>
<td>13 ( 12 (3 - y) = z )</td>
<td>( y = -3 )</td>
<td>( z = ? )</td>
<td>D 0</td>
</tr>
</tbody>
</table>

### Why is a witch like a candle?

| 4 | 3 | 1 | 6 | 5 | 2 | 1 | 8 | 9 | 4 | 3 | 13 | 10 | 12 | 11 | 1 | 7 |

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Application A:
Material required: Graph paper (1mm or 2mm)

The School Social

John and his friend have been given the job of organising the entertainment for the school social. After some research they have come up with the following list of alternatives:

- A comedian who charges a $40 appearance fee plus $75 per hour.
- A band which charges a $200 appearance fee but only $25 per hour.
- A disc jockey who charges $50 per hour. However the school must provide the sound system, which will cost $150 to hire for the evening.

Before deciding, John thought it would be a good idea to work out the cost of each of the alternatives as a function of the length of the social (which had not been finalised).

Task 1: Fill in the table below to show the cost of each alternative for 1 to 5 hours.

<table>
<thead>
<tr>
<th>Length of social (hours)</th>
<th>t</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of comedian ($)</td>
<td>c</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost of band ($)</td>
<td>b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost of disc jockey ($)</td>
<td>d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Task 2: Which is the cheapest alternative if the social is 3 hours long?

Task 3: Which is the cheapest alternative if the social is 4 hours long?

Task 4: Comment on the various costs for running a 2 hour social.
John’s friend Jeanette feels that they might make easier comparisons if they were to produce a graph showing the relative costs.

**Task 5:** On a sheet of graph paper, carefully choose an appropriate scale and then plot the points from the table above. For each one, draw a straight line through the points. Label each line and label the axes.

**Task 6:** Use your graph to complete the following:

(a) For how long, to the nearest hour, is the comedian the cheapest alternative?

(b) Complete the sentence: “Between 2 and 4 hours the most expensive option is _______________”

(c) At two hours, which two alternatives cost the same?

(d) At approximately 3 hours it costs the same to hire the __________ as it does the __________.

(e) At what time exactly are the costs in part (d) the same?

(f) Find the time for which the costs are the same for the comedian and the disc jockey.

**Task 7:** Find a linear function for each of the three alternatives.

\[ c = \_\_\_\_ t + \_\_\_\_ \]

\[ b = \_\_\_\_ t + \_\_\_\_ \]

\[ d = \_\_\_\_ t + \_\_\_\_ \]

**Task 8:** Use these functions and substitution to check your answer to part (f) of task 6. If it is not correct then use guess, check and improve methods to find the answer more accurately.

\[ \text{Time} = \_\_\_\_ h \]

\[ = \_\_\_\_ h \_\_\_\_ \text{min} \]
Application B:
Material required: Graph paper (1mm or 2mm)

A Cheaper Taxi Ride

Customers who use the “Ride Easy” taxi company are offered the following choice of fare schedules.

- System A: An accumulating fee of $1.25 per kilometre.
- System B: A flag fall (fixed fee) of $3.50 and an accumulating fee of 80c per kilometre. The flag fall is charged as you start your journey.

This application looks at the relative costs of the two systems. You must decide which is the better system for short distances and which is the better system for longer distances. You must also decide at what point a “short” distance becomes a “longer” distance.

Task 1: Start by creating a table of values in the form below:

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>System A ($)</th>
<th>System B ($)</th>
</tr>
</thead>
</table>

From your table you should now have a rough idea of the answers to the question you are investigating.

Task 2: Use graph paper to plot an accurate pair of lines representing the two systems.

Task 3: What feature on the graph represents the point where a “short” distance becomes a “longer” distance?

Task 4: What will the costs be at that point?

Task 5: Use your graph to estimate the point at which a “short” distance becomes a “longer” distance, then use guess, check and improve to find the answer accurate to 1 decimal place. Report below on your findings.
Application C:
Material required: Graph paper (1mm or 2mm)

Draining Quietly

Charlie has built a new 34 kilolitre capacity tank on his farm to hold water for his pigs. Unknown to Charlie there was a steady leak in the tank which he has only recently discovered. In order to evaluate his chances of making it through the summer Charlie has made some measurements of the rate of leakage.

Charlie began his measurements one month after completing and initially filling the tank to capacity (i.e. at the end of week 4). At that time the tank contained 30.8 kL. Two weeks later he measured it again and found that it now contained 29.2 kL.

Task 1: Use this information to complete the table below. Assume a steady loss of water.

<table>
<thead>
<tr>
<th>End of week (n):</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of water left in kL (w):</td>
<td>30.8</td>
<td></td>
<td>29.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Task 2: How much water, in kilolitres, is the tank losing per week?

Task 3: How much water will be in the tank at the end of:

Week 12: Week 20:

Task 4: Write a linear function giving the amount of water left (w) from the week number (n). Use your function to check your answers to task 3, and to complete the tasks which follow.
Task 5: Using this **linear function** requires some assumptions about the leak. Write about any assumptions that may have been necessary.

Task 6: How much water was there in the tank initially, according to your rule?

Task 7: How many weeks will it be before the tank is empty?

Task 8: Your **rule**, when graphed, will be a straight line. Will the **gradient** of this line be positive or negative?

Task 9: What physical reason is there for the sign of the **gradient**? i.e. Knowing what you do about the situation, what would have told you that the **gradient** would be negative even before finding the **rule**?

Task 10: Imagine a bucket with a small hole in the bottom through which water is escaping. If you were to graph the level of water in the bucket against time, would the graph be a straight line? If not, what shape might it be. Draw a graph of the water level over time on axes like the ones below. Explain your reasons.
Application D:
Materials required: Graph paper (1mm or 2mm)

Catering for the Masses

Fergus is planning to open a catering business and needs to determine what charges to use. He must compete with two main established firms and has been gathering information on them.

- **Occasional Foods.** This firm aims for the “entertainment while dining” market and provides entertainment during the meal in the form of a clown, a singer or a small instrumental group. This means that they charge quite a large fixed fee and so their costs are high for small groups. To offset this they don’t charge much per person and so their market is largely very big gatherings such as children’s birthdays (the clown), engagement parties or weddings.

- **Intimate Dining.** This firm is more up-market and caters for small gatherings such as dinner parties, adult birthdays and barbecues. They don’t provide any entertainment but they do provide high quality food and waiters and waitresses to serve it – the more guests, the more waiters. Their charge per guest is much higher because of this.

Fergus intends aiming for the middle ground. He is not interested in providing entertainment but he is a chef and is proud of his food. He wants to provide a quality product but still wants to be able to compete with both firms for the size of party he is aiming at. To better understand their charges, he phones each firm and asks them what the cost would be for a party of five. His wife then phones them and asks for their prices for a party of twenty people. The results of this are shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Five people</th>
<th>Twenty people</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occasional Foods</td>
<td>$330</td>
<td>$480</td>
</tr>
<tr>
<td>Intimate Dining</td>
<td>$195</td>
<td>$570</td>
</tr>
</tbody>
</table>

**Task 1:** Assuming that each firm uses a linear function to determine its charges, find rules giving the charges in the form:

\[
C_{OF} = _____ p + _____
\]

\[
C_{ID} = _____ p + _____, \text{ where}
\]

p is the number of people and \( C_{OF} \) and \( C_{ID} \) are the charges for each firm.
Task 2: For each rule, explain how the values it contains reflect the aims of the company concerned. For example, what part of the rule for Occasional Foods reflects their aim of keeping the cost per person down, and what part their provision of entertainment?

Task 3: Using guess, check and improve, or a calculator, decide for what range of values Intimate Dining is cheaper than Occasional Foods and vice versa.

Task 4: Using a scale as shown right, graph the charges for each company against the number of people on the same set of axes. Use your graph to check your answer to task 3.

Task 5: Fergus has decided that he will aim at parties that contain between 10 and 25 people so he must ensure that his charges are cheaper (or at least the same) for all values from 10 to 25 inclusive. He still wants to use a linear function. What should it be? Note: There is more than one answer which will work but there is only one which gives the cheapest charges for the range of people he is interested in.

Task 6: Graph Fergus’ charging rule on the same axes as you used before. Explain in words below what features of the graph reflect the way Fergus’ rule works.
Application E:
Materials required: Computer and Spreadsheet File

Spreadsheet Formulae

Task 1:  Open the spreadsheet file
"D02_AppE_Spreadsheet_Formulae.xls" and click
on the tab at the bottom of the page titled
"Introduction".  Read this introductory page.

Task 2:  Now click on the tab "Sheet 1" to bring up the Distance Conversion
Spreadsheet.  Read through this to get an idea of what it does.

Task 3:  By typing in the values below into any of the cells from B5 to Z5 convert
these distances in kilometres to distances in miles.

(a)  5km  (b)  100km  (c)  60km

(d)  22.56km  (e)  0.895km  (f)  2150km

Task 4:  Now use the spreadsheet to complete two conversions from kilometres to
miles of distances of your choice.

(a)  __________ km = __________ miles

(b)  __________ km = __________ miles

Task 5:  To how many decimal places does the spreadsheet give the values in miles?

Task 6:  Click on cell B6.  The Name Box above the spreadsheet will read B6 and at
the same time the Formula Bar to the right (fx) will show the formula that
is entered into all of the cells from B6 to Z6.  Write down what is shown in
the Formula Bar.

Task 7:  Explain how this formula relates to the function: \( m = \frac{k}{1.609} \)
Task 8: By typing these values into any of the cells B11 to Z11 convert these distances from miles to kilometres:

(a) 10 miles   (b) 65 miles   (c) 450 miles

Task 9: Also write two of your own conversions below:

Task 10: How many decimal places are shown for the kilometres values?

Task 11: Click on the cell B12 and write down what is shown in the formula bar.

Task 12: Explain how this relates to the function: \( k = 1.609 \times m \)

Task 13: Now click on the tab "Sheet 2" to bring up the Circumference of a Circle Spreadsheet. Read through this to get an idea of what it does.

Task 14: By typing the values below into the correct cells complete these calculations.

(a) Radius = 15mm, Circumference =  
(b) Radius = 4.5cm, Circumference =  
(c) Diameter = 32m, Circumference =  
(d) Circumference = 480mm, Radius = , Diameter =  

Task 15: Complete four calculations of your own:

Task 16: How many decimal places are given for values in this spreadsheet?

Task 17: Write down the formulae in cells B7, B10, B17 and B18.

Task 18: Explain how these formulae relate to the functions given at the bottom of the spreadsheet.

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Task 19: Now click on the tab "Sheet 3" to bring up the Temperature Conversion Spreadsheet. Read through this to get an idea of what it does.

Task 20: By typing the values below into the correct cells complete these conversions.

(a) $100^\circ C = \_\_\_\_\_\_\_^\circ F$
(b) $38^\circ C = \_\_\_\_\_\_\_^\circ F$
(c) $32^\circ F = \_\_\_\_\_\_\_^\circ C$
(d) $50^\circ F = \_\_\_\_\_\_\_^\circ C$

Task 21: Complete four conversions of your own:

Task 22: How many decimal places are given for values in this spreadsheet?

Task 23: Write down the formulae in cells B9 and B15.

Task 24: Explain how these formula relate to the functions given at the bottom of the spreadsheet.

Task 25: Why do cells B9 to Z9 start with $32^\circ F$?

Task 26: Why do cells B15 to Z15 start with $-18^\circ C$?

In Application D Fergus decided that the function he should use to charge people was:
\[ c = 14p + 180 \] (where $c$ was the cost in $ and $p$ was the number of people)

Task 27: Now click on the tab "Sheet 4" to bring up the Fergus' Catering Spreadsheet. Read through this to get an idea of what it does.

Task 28: Click on cell C17 and then type: $=C16*14+180$ and then Enter. (Make sure you include the equal sign, this shows it is a formula.)

Task 29: Why does cell C17 immediately show 180?

Task 30: Click on cell C17 again and then from the Menu Bar select Format and Cells. Under the Number tab click on Currency then click OK. What effect does this have?

Task 31: Use the spreadsheet to calculate what Fergus should charge for:
(a) 17 people
(b) 23 people
Application F:
Materials required: Newspaper

Newspaper Search:
Holiday or Work?

Task 1: The clipping shown right shows the cost of a tour of the South-West.

1. What will be the total cost of a tour for five people?

2. Complete this table of costs for various numbers of people.

<table>
<thead>
<tr>
<th>No. of people</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost of tour</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Draw a graph to represent the costs for various numbers of people.

Task 2: This clipping shows the cost of staying at the Green Island Lodge.

4. What will be the total cost of a one night stay for two people? For four people?

5. Explain why the cost might not be $360 for three people.

6. James called up to find out what “Extra nights at reduced rates” meant and was told that subsequent nights were charged at $100 per person, per night, twin share. Why might the cost be less for subsequent nights? Hint: Look at what the “Offer - Includes”

7. Find a function which will give the cost per person, twin share, for any number of nights.

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**Task 3:** The advertisement shown here gives the cost of brick paving.

8. What does per sqm mean?

9. If it is possible to buy part of a square metre, what will be the cost of paving:
   (a) $1.5 \text{ m}^2$?
   (b) $2.4\text{ m}^2$?
   (c) $2.9\text{ m}^2$?

Further investigation shows that you can buy the pavers in packs of 100, with each pack covering 3 square metres. The cost of a pack of 100 is $25.

10. Assume that you would buy part of the area as a pack of 100 whenever possible. What will be the cost of paving:
     (a) $3\text{ m}^2$?
     (b) $5\text{ m}^2$?

11. What do you notice about the cost of $3\text{ m}^2$ versus the cost of $2.9\text{ m}^2$? Why does this happen? For what range of values between $2\text{ m}^2$ and $3\text{ m}^2$ is it cheaper to buy a pack instead of by the square metre?

12. Draw a graph showing the cost versus area to be paved for zero to twelve square metres. Note that this graph will be greatly complicated by the lower cost of buying in packs of 100 – you will need to be careful near $3\text{ m}^2$, $6\text{ m}^2$, $9\text{ m}^2$ and $12\text{ m}^2$. You may wish to discuss the graph with your teacher if you are not sure what this means.

13. Discuss why it would be difficult to find a rule for this situation. When you have an opinion consult your teacher.

**Task 4:** Use a newspaper to find an example of a situation which can be modelled using a linear function. As in the examples above, you can use your imagination to provide “extra” information for your advertisements. In each case you should either find a function for your situation or draw a neat, labelled graph if the rule is difficult to find.
Application G:
Materials required: Newspapers, brochures, Internet access

Project: Mobile Phones

Contracts for mobile phones are usually very complex. To attract customers to sign up with them companies advertise free phones and lots of other special deals. Of course all of these deals are such that the customer ends up paying in one way or another.

The charging methods for these contracts involve linear relationships dependant on the phone supplied, the length of the contract and the number and duration of calls made.

Task 1: Collect advertisements for mobile phones from five different companies. These can be from newspapers, the Internet, brochures or television.

Task 2: Read through the advertisement and the fine print and write out the pricing details for each contract. Include any costs for leaving the contract and the minimum charge where it is shown.

Task 3: Write down a standard set of conditions so that you can compare different contracts. For example; A Nokia 3310 phone, 24 months contract, 30 calls per month totalling 60 minutes call time.

Task 4: Find the cost of phones from each of your advertisements for the standard set of conditions you set up in task 3. This may require you to source extra information by getting brochures sent to you, ringing the companies sales number, using the Internet or visiting their sales outlet.

Task 5: Find out the cost of a home phone for the same conditions. Your parents should be able to provide you with the costs of a home phone.
Application H:
Materials required: Internet access

Project: Online Learning

The Internet has a lot of web sites built to assist students learning. Some of these sites contain materials on Linear Functions. This project will enable you to expand your learning through the use of these web sites.

Task 1: Visit the following web page for this Application:

Task 2: Follow the “Links to sites on Linear Functions” to at least three of these sites. Spend from 5 to 20 minutes at the site reading the materials and completing the tasks.

Task 3: Make notes about what you found at each of these sites. Group these notes under two headings:
1. New information and concepts
2. Information covered in this module.

Task 4: Using the Direct Searches or the Search Engine links at the bottom of the web page or your own Search Engine links find at least three more sites with useful information on Linear Functions. Write down the address (URL) of these sites with a description of what these sites contain.

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**Student Reflection**

What have I learned in this module?

What new words did I learn during this module?

Look at the outcomes at the start of the module (page 3). Have I progressed on each of these outcomes?

What do I need to improve on?

Write about one thing in this module I found interesting.

What do I think was the most important concept in this module?

Where could the maths in this module be used in our society?

One area I would like to look more at is:

Write something about how the bits in this module connect to each other.

Write something about how the bits in this module connect to other modules.
ANSWERS TO EXERCISES:
In all Exercises drawings or sketches should be checked with another student or your teacher.

Exercise 1:
1. 7, 9, 11, 13, 15 - linear  
2. 7, 10, 13, 16, 19, 22 - linear  
3. 3, 7, 11, 15, 19, 23, 27 - linear  
4. 3, 6, 9, 12, 15, 18, 21 - linear  
5. 0, 1, 2, 3, 4, 5, 6 - linear  
6. 1, 4, 9, 16, 25, 36, 49 - NOT linear  
7. 1, 5, 9, 13, 17, 21, 25 - linear  
8. 2, 4, 8, 14, 22, 32, 44 - NOT linear

Exercise 2:
1. 3, 4, 5, 6, 7, 8, 13  
2. 1, 4, 9, 16, 25, 36, 49 - NOT linear  
3. 3, 4.75, 5, 5.5, 6.25, 6.875, 7.5  
4. 18, 14, 10, 6, 0, -5, -10  
5. -2, 0, 6, 8, 16, 21, 26  
6. The independent variable does not increase in a linear pattern.

Exercise 3:
1. 10 - linear  
2. 13 - linear  
3. 20 - linear  
4. 2 - linear  
5. 25 - NOT linear  
6. 6 - NOT linear  
7. 19 - linear  
8. 32 - NOT linear  
9. 17 - linear  
10. 6 - NOT linear  
11. 12 - NOT linear  
12. 5 - NOT linear  
13. 69 - linear  
14. 18 - NOT linear  
15. 5 - NOT linear  
16. 125 - NOT linear

Exercise 4:
1. \( b = a + 5 \)  
2. \( d = c - 2 \)  
3. \( f = 3e \)  
4. \( h = 2g + 1 \)  
5. \( n = 4m - 2 \)  
6. \( q = 10 - p \)  
7. \( s = r/10 \)  
8. \( y = 3x + 2 \)  
9. \( f(x) = 0.5x + 4 \)  
10. \( f(x) = -2x - 3 \)  
11. \( f(x) = 22 - 3x \)  
12. \( f(x) = 4x + 3 \)

Exercise 5:
1.  
2.  
3.  
4.  
5. All four graphs are lines
Exercise 6:

1. Increases the gradient of the lines.

2. Same gradient and are parallel.

3. Both have the coefficient of $x = 3$

4. Parallel and are sloping downwards.

5. Slope down at a gradient of 2.

6. They have the same coefficient of $x$ because $0.5 = x/2$.

7. The scales on the axes are different.

8. This causes the graphs to appear to be different. However, they have the same gradient and $y$ intercept.

9. It moves the graph up and down.

10. i.e. it changes the $y$ intercept
**Exercise 7:**
1. gradient 1, y intercept 3
2. gradient 2, y intercept -3
3. gradient -2, y intercept 7
4. gradient 4, y intercept 1
5. gradient -3, y intercept 5
6. gradient 2, y intercept 5
7. gradient 1/4, y intercept -5
8. gradient -0.5, y intercept 6
9. gradient -2, y intercept 7
10. gradient 2, y intercept -1, function $y = 2x - 1$
11. gradient 5, y intercept 2, function $y = 5x + 2$
12. gradient -2, y intercept 10, function $y = -2x + 10$ or $y = 10 - 2x$
13. gradient 0.5, y intercept 3, function $y = 0.5x + 3$
14. gradient 3, y intercept -4, function $y = 3x - 4$
15. gradient -4, y intercept 7, function $y = -4x + 7$ or $y = 7 - 4x$

**Exercise 8:**
1. (a) 9 (b) 9 (c) 19
2. (a) and (b) are equivalent
3. (a) 14 (b) 14 (c) 22 (d) 18 (e) 14 (f) 12
4. (a), (b) and (e) are equivalent
5. (a) 18 (b) -14 (c) 12 (d) 18 (e) 18 (f) 20
6. (a), (d) and (e) are equivalent
7. (a) 23 (b) 42 (c) 23 (d) 22 (e) 23 (f) 23
8. (a), (c), (e) and (f) are equivalent
9. (a) -28 (b) -112 (c) -28 (d) 28 (e) -28 (f) -28
10. (a), (c), (e) and (f) are equivalent

**Exercise 9:**
1. (a) True (b) False (c) True (d) False (e) True (f) True
2. (a) 5 (b) 12 (c) 4 (d) 9 (e) -5 (f) 10
3. (a) 6 (b) 6 (c) -4 (d) -10 (e) 8 (f) 20
4. (a) 5 (b) 6 (c) 6 (d) 0 (e) 2 (f) 1.5
5. (a) 2 (b) 7 (c) 8 (d) -1 (e) 6 (f) 4
   (g) 3 (h) -1.3 (i) 5 (j) -3.5

**Exercise 10:**
1. (a) 3 (b) 4 (c) 7 (d) 1 (e) 5 (f) 2 (g) 8 (h) 6
2. (a) $8a + 2$ (b) $6b - 7$
   (c) $5c + 2$ (d) $5(d + 2)$ or $(d + 2) \times 5$
3. (a) $x + 8 = 15, x = 7$ (b) $3x = 21, x = 7$
   (c) $x - 7 = 5, x = 12$
   (d) $\frac{X}{4} = 5, x = 20$ (e) $2x + 3 = -5, x = -4$