INVESTIGATION # 12

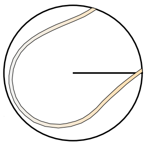
Exponential Models

In some real applications the formula or model that determines values for various situations is governed by an exponential function. This investigation looks at these Exponential Models.

Model 1: A Bouncing Ball

When a ball is dropped it bounces up to a fraction of its previous height. If, for example, this fraction was three quarters of its previous height then the exponential model would be a function like $b = h.(0.75)^n$ where h is the height the ball was dropped from and b is the height of the nth bounce.

For example:



A ball is dropped from 2m and bounces to three quarters of its previous height on each bounce.

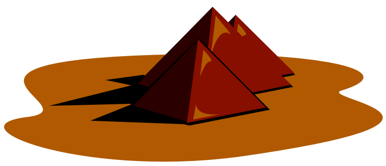
The function would be: $b = 2 \,.\, (0.75)^n$

Its first bounce would be $2 \times (0.75)^1 = 1.5$m high.

<u>Task #1</u>

(a) Work out what height its next 4 bounces would be.
(b) Draw a graph of these values, n on the horizontal axis, b on the vertical axis.
(c) What height would its 10th bounce be?
(d) Write the function for a ball dropped from 5m that bounces up to four fifths its previous height.
(e) How high would this ball bounce to on its 10th bounce?

Model 2: Painting The Pyramids



One theory on how the Egyptians got light into the inner chambers of their pyramids to paint them was by using a series of mirrors along the tunnels.

The amount of light reflected by a series of mirrors can also be modelled by an exponential function.

If each mirror was to reflect 50% of the light that falls on it then amount (R%) the nth mirror would reflect would be given by:

$$R = 100. \ (0.5)^n$$

For example:

If there was a series of three mirrors then the amount of light reflected by the third mirror would be:

$$R = 100 \ . \ (0.5)^3 = 12.5\% \text{ of the original light.}$$

Task #2

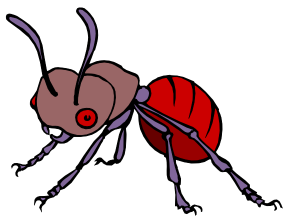
(a) In one pyramid there would need to be a series of nine mirrors. If these mirrors reflected 50% of the light how much would reflect from the ninth mirror?

(b) If 40% is a better estimate of the reflectivity of mirrors in those days how much light would then reflect from the ninth mirror?

Model 3: Population increase

The number of organisms in a population that grows without any restrictions can also be modelled by an exponential function.

For example:



If the amount of ants in a colony increases by 8% every year then the number of ants (A_t) in a colony after t years, given that it started with A_0 ants, would be given by the function:

$$A_t = A_0 \ . \ (1.08)^t$$

Hence, after 5 years a colony of 1000 ants would increase to about 1470 ants.

Task #3

(a) How many ants would be in this colony after 20 years?

(b) Draw a graph showing the increase in the number of ants in this colony from the original 1000 for the first 20 years.

(c) How many years would it take for this colony to have more than 10 000 ants?

(d) If another colony of 1000 ants increases at 15% every year compare how the amount of ants in this colony increases.

ASSESSMENT TASK

Exponential Models

1. Explain in a few sentences what you understand by the term “Exponential models”.

2. Given the model: $V = 0{\cdot}85^n$ complete this table of values:

n	1	2	3	4	10
V					

3. The amount of money in an investment follows the same model as the examples in the investigation. If the model for the amount of money (A) a \$2000 investment accumulates to after t years is given by:

$$A = 2000\,(1{\cdot}07)^t$$

complete this table showing the amount after various years.

Years (t)	Amount (A)
1	
2	
5	
10	
50	

4. The amount of bacteria in a colony also follows an Exponential model. If the number of bacteria (N) in a colony after t days is given by the model:

$$N = 50\,(1{\cdot}8)^{t}$$

draw a graph showing how the population increases over a 20 day period.

5. A new improved domestic dry cleaning machine was made that lost only 1% of the dry cleaning liquid every time it was used. If Q is the amount started with and n is the number of times it is used what would the model be for the amount left (L)?

L = ____________________