

INVESTIGATION # 19

Proportion Functions

In situations involving proportion there is always some underlying function that can be used to determine unknown values.

In the case of direct proportion the underlying function is a linear function of the form:

$$y = k.x$$

Here is a simple example of how this function can be used to solve problems in direct proportion:

Sixty text books cost the maths department \$256.20.

- (a) How much would 25 cost?
- (b) How many could be bought for \$213.50?

Since this is direct proportion we use a linear function $c = k.b$ where c is cost and b is number of books

We use the information given to find k :

$$\begin{aligned} 256.20 &= k \cdot 60 \\ k &= \frac{256.20}{60} \\ k &= 4.27 \end{aligned}$$

We then write down the full linear function for this question: $c = 4.27b$

(In this problem - what does the k value represent?)

Now we use that to answer the questions:

- (a) $c = 4.27b$
 $c = 4.27 \times 25$
 $c = 106.75$ i.e. \$106.75
- (b) $c = 4.27b$
 $213.50 = 4.27b$
 $b = 50$ i.e. 50 books

For direct proportion that involves a linear function there are probably easier ways

to do the question but this method works for all types of proportion.

Now lets look at a more complex proportion situation solved the same way.

The brightness of a light is inversely proportional to the square of the distance from that light. A light has a brightness of 40 candela at a distance of 1.5 metres.

- (a) find the constant of proportion (k)
- (b) what will the brightness be at a distance of 2 metres ?
- (c) at what distance will the brightness be reduced to 5 candela ?

In this case the underlying function is $y = \frac{k}{x^2}$ so we will set up the function as:

$$b = \frac{k}{d^2} \quad \text{Where } b \text{ is the brightness and } d \text{ is the distance.}$$

Step 1 again: Use the information to find k:

$$40 = \frac{k}{1.5^2}$$

I.e

$$k = 40 \times 1.5^2$$
$$k = 90$$

Step 2 again: Write down the full function: $b = \frac{90}{d^2}$

Now answer the questions:

- (a) the constant of proportion is 90
- (b) $b = \frac{90}{2^2}$ i.e. $b = 22.5$ i.e. 22.5 candela
- (c) $5 = \frac{90}{d^2}$ i.e. $d^2 = \frac{90}{5}$ i.e. $d^2 = 18$ i.e. $d = 4.24\text{m}$

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Your tasks are:

Task #1: Follow through the two examples given above and understand the process.

Task #2: Use this method with the proportion functions given in these questions to solve the problems.

1. The pressure exerted on an object in a liquid varies directly with the depth of that object (model is $y = kx$). A ball bearing immersed to a depth of 20 cm in oil has a pressure of 1.8 kilopascals (kPa) exerted on it. Find:

- (a) the pressure on this ball bearing if it is at a depth of 75 cm
- (b) the depth that will produce a pressure of 18 Kpa on the bearing

2. The volume that a sample of gas will take up is inversely proportional to its pressure when temperature is constant (model is $y = \frac{k}{x}$). If 500 cm³ of a gas sample is at 300 mm pressure;

- (a) find the proportion constant
- (b) what will the volume be at 500 mm pressure ?
- (c) what pressure will be exerted if the volume is increased to 750 cm³ ?

3. The area of a circle (a), is directly proportional to the square of its radius (r) (model is $y = kx^2$). If a circle with radius 5cm has an area of 78.5 cm² complete the proportion equation given below by finding k.

$$a = k.r^2$$

4. The frequency of a stretched string is inversely proportional to the square root of the wire's mass (model is $y = \frac{k}{\sqrt{x}}$). If a wire of mass 5 grams gives a frequency of 100 Hertz (Hz) find
- (a) the proportion constant
 - (b) the frequency of a 2g wire
 - (c) the wire mass required to give a frequency of 200 Hz.

Task #3: Find the appropriate proportion function for this question and use it to solve the problem.

The time taken to travel by car from Perth to Margaret River is inversely proportional to the speed at which the car travels. If the journey takes $2\frac{1}{2}$ hours at 110 km/h:

- (a) set up an equation for the inverse proportion and calculate the proportion constant
- (b) what does this proportion constant represent ?
- (c) how long will the journey take at 100 km/h ?
- (d) if an ambulance does the journey in 2 hours what was its average speed ?

ASSESSMENT TASK

Proportion Functions

1. A physical property of elements known as density is based on the fact that the mass of any substance is directly proportional to its volume ($m = kv$).

(a) Calculate the proportion constants (densities) for the following substances from the mass and volume given.

(i) 50g of Aluminium has a volume of 18.5 cm^3

(ii) 250 g of Copper has a volume of 27.9 cm^3

(iii) 5000 cm^3 of Neon gas a mass of 4.195g.

(b) Use the constant calculated in (ii) to find the mass of 1 cubic metre of copper.

2. An electric bar heater gives out 450 kilojoules (kJ) of heat (h) when it is run on a 10 amp current (i). If the formula for heat from an electric current is:

$$h = k.i^2$$

find the proportion constant from the given information and use it to calculate the heat that this heater would produce on a 15 amp current.

3. Triple J FM radio station broadcasts radio waves 3.0212 metres long at a frequency of 99.3 megahertz (MHz). If the frequency of radio waves is inversely proportional to their wavelength ($f = \frac{k}{w}$);
- (a) find the length of radio waves from PM FM which broadcasts at 92.9 Mhz
- (b) which FM station has radio waves of length 3.1217 m ?
4. The frequency of the sound produced by a guitar string is directly proportional to the square root of its tension. If a tension of 20 Newtons produces a sound with a frequency of 40 Hertz what frequency will be produced when the string's tension drops to 15 Newtons?
5. The volume of a sphere is directly proportional to the cube of its radius. A sphere with radius 3.5mm has a volume of 179.6 mm³. Find:
- (a) the volume of a sphere with radius 6.2cm.
- (b) the radius of a ball with volume 500 cm³.
- (c) what does the value of k represent in this proportion?